The size of the instabilities in the direction of SW propagation is evidently on the same order as the transverse size of the compressed arc. Therefore, a decrease in the initial size of the arc should lead to higher quenching parameters, which was observed in the experiments with a cylindrical channel.

The instabilities developing at the contact surface lead to a considerable increase in its area, which increases the energy flux due to radiant heat transfer and promotes the fine-scale mixing of detonation products with the arc plasma, its cooling, and decay. As the estimates show, the detonation products in a cylindrical channel can absorb about 600 J without a change in their properties.

The authors thank A. P. Ershov, A. L. Kupershtokh, and A. A. Kuzovnikov for useful discussions and help in conducting some of the experiments.

## LITERATURE CITED

1. P. I. Zubkov, L. A. Luk'yanchikov, and K. A. Ten, "On the quenching of an electric arc with explosives," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1978).
2. E. E. Meshkov, "Instability of the interface between two gases being accelerated by a shock wave," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1969).
3. V. S. Sizonov, "Behavior of the interface between two fluids of finite depth in variable force fields," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1970).

# PRESSURE DEPENDENCE OF ELECTRICAL CONDUCTIVITY IN HIGH MAGNETIC FIELDS

B. A. Boiko and S. P. Roshchupkin UDC 538.6

Obtaining pulsed high magnetic fields by flux compression in an explosive-driven metal shell is limited primarily by two processes: the dynamics of the compression and the diffusion of the field, related to the finite electrical conductivity $\sigma$ of the liner. The character of the dependence of the electrical conductivity on the physical properties of the medium (the density $\rho$ and the specific thermal energy q) significantly affects the results of theoretical calculations [1-3]. The dependence of the electrical conductivity of metals on q and $\rho$ has not been adequately studied for high temperatures and high and low densities. Therefore such semiempirical relations as [3, 4]

$$\sigma = \sigma_0(1 + \beta q)^{-1} \ [2], \ \sigma = \sigma_0(1 + \beta q)^{-1}(\rho/\rho_0)^n$$

are generally extrapolated into the range of densities and temperatures which are developed in a medium when high magnetic fields are produced. Numerical calculations in [2] made under the assumption that the electrical conductivity of copper depends only on Joule heating are in good agreement with experiment for fields $H \leq 3$ MOe. Numerical calculations of theoretical upper bounds of magnetic fields presented in [3] agree with experiment for $H > 10$ MOe only when account is taken of the dependence of the electrical conductivity of the shell on both Joule heat and density in the region of compression. A relatively weak dependence of the maximum value of the field on Joule heat is noted. Thus, an arbitrary change of the heat coefficient $\beta$ by a factor of three did not lead to better agreement of the calculated results [3] with experiment. Fields $H > 10$ MOe exert pressures $p > 4 \cdot 10^{11}$ Pa on a conductor, which leads to an increase in density, and consequently to an increase of the electrical conductivity of certain metals [5, 6]. Estimates show that the change of the electrical conductivity as a result of heating is more than an order of magnitude larger than the change due to pressure. Therefore, taking account of the above, it is of interest to find out in which cases a correction to the change of the electrical conductivity as a result of pressure may turn out to be substantial. The investigation of the penetration of high magnetic fields into a conductor is complicated by nonlinear effects related to the decrease of its electrical conductivity during heating and vaporization and the increase in density under increased pressure, and is possible in general only by numerical methods. We note that numerical calculations do not permit an estimate of qualitative regularities. An analytic solution which takes account of the fundamental physical processes, even if it is obtained by greatly simplifying the problem, gives a deeper understanding of the phenomena,

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 134-139, September-October, 1981. Original article submitted July 7, 1980.

 0021-8944/81/2205-0706$07.50 

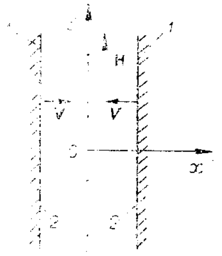

Fig. 1

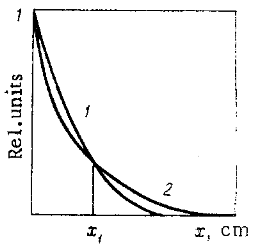

Fig. 2

and facilitates subsequent more accurate computer calculations. In view of this, we leave aside the question of the dynamical limitations, and assuming that certain parameters of the shell (electrical conductivity, equation of state) are known, we consider the problem of the effect of pressure on the magnitude of the magnetic field during its compression by the metal walls of a liner, and present qualitative analytical relations which permit an estimate of its effect.

1. We consider the one-dimensional case of the compression of magnetic flux by two plane metal walls (Fig. 1, where 1 is a metal shell moving with velocity **v**, 2 is the surface of the shell, and **H** is the magnetic field in the gap, directed along the y axis). The flux balance equation has the form

$$\Phi_0/2 = \Phi_1(t) + \Phi_2(t),$$

where $\Phi_0$ is the initial flux; $\Phi_1(t) = H_0(t)x(t)$ is half the flux between the walls; $\Phi_2(t) = \int_0^\infty H\,dx = H_0(t)\,s(t)$ is the flux in the wall. Hence, for the magnetic field between the walls we have

$$H_0(t) = \frac{\Phi_0/2}{x(t) + s(t)}. \qquad (1.1)$$

We consider this expression at the instant the liner stops. We denote by $x_*$ the distance from the y axis to the reversal point. It follows from Eq. (1.1) that for $x_* \lesssim s$ the correction to the depth of the skin layer s because of pressure can be substantial. Let $H_1$ be the magnetic field calculated from (1.1) under the assumption that the electrical conductivity of the liner depends only on Joule heat [s(t) is the depth of the thermal skin layer], $H_2$ is the analogous quantity for a medium whose electrical conductivity depends on both Joule heat and pressure [$s(\tau_2)$ is the depth of the skin layer in this case]. The ratio of these fields at the same distance from the reversal point is

$$\frac{H_2}{H_1} = \frac{1 + x_*/s(t)}{s(\tau_2)/s(t) + x_*/s(t)}. \qquad (1.2)$$

It follows from (1.2) that to estimate the effect of pressure on the magnitude of the magnetic field it is sufficient to determine the ratio of the corresponding skin-layer thicknesses.

In the one-dimensional case under consideration of the penetration of a plane magnetic field into the half-space $x > 0$, the diffusion and heat-conduction equations have the form

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\frac{c^2}{4\pi\sigma}\frac{\partial H}{\partial x}; \qquad (1.3)$$

$$\frac{\partial q}{\partial t} = \frac{c^2}{16\pi^2\sigma}\left(\frac{\partial H}{\partial x}\right)^2. \qquad (1.4)$$

We assume that the material of the conductor exists in only two states:

1) a completely conducting metal whose electrical conductivity is [3, 4]

$$\sigma = \sigma_0(1 + \beta q)^{-1}(\rho/\rho_0)^n, \qquad (1.5)$$

where $\sigma_0$ and $\rho_0$ are, respectively, the electrical conductivity and density of the medium at 20°C; $\beta$ is the heat coefficient; q is the thermal energy density; n is a factor depending on the medium, which is equal to 2.7 for Cu and 6 for Pb [5];

2) an expanded nonconducting vapor ($\sigma = 0$).

This assumption is based on the results of [3] where it was shown that the electrical conductivity of a plasma can be neglected.

We use the following equation of state [7]:

$$p = A[(\rho/\rho_0)^m - 1], \tag{1.6}$$

which is valid in the region of cold compression for pressures $p \lesssim 10^{11}$ Pa, where m and A are constant parameters equal to 4.8, 5.3 and $3.05 \cdot 10^{10}$ Pa, $0.87 \cdot 10^{10}$ Pa for Cu and Pb, respectively.

The pressure of the magnetic field for an incompressible conductor is [4]:

$$p(x, t) = [H^2(0, t) - H^2(x, t)]/8\pi. \tag{1.7}$$

Taking account of Eqs. (1.5)-(1.7), we write the electrical conductivity in the form

$$\sigma = \sigma_0 [1 + \beta q(x, t)]^{-1} \left[1 + \frac{H^2(0, t)}{8\pi A} - \frac{H^2(x, t)}{8\pi A}\right]^\delta, \tag{1.8}$$

where $\delta = n/m$.

2. The system of partial differential equations (1.3), (1.4) with condition (1.8) in general can only be solved numerically. Therefore, in order to make the necessary qualitative estimates, we employ the condition that the variation of the electrical conductivity of the medium with magnetic pressure is independent of the variation with Joule heat (1.8), and also the fact that the velocity of the pressure pulse is more than an order of magnitude higher than the rate of diffusion of the magnetic field [4]; i.e., diffusion occurs in compressed cold metal. Thus, to start with, we consider the penetration of the field into a medium whose electrical conductivity depends only on pressure, neglecting Joule heat. This enables us to show that the effect of pressure on the electrical conductivity is reduced to its increase over the whole skin layer by a certain number of times.

Under these assumptions the initial equation has the form

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x} \frac{c^2}{4\pi\sigma} \frac{\partial H}{\partial x}; \tag{2.1}$$

$$H(0, t) = H_0(t), \; H(x, 0) = 0, \tag{2.2}$$

where

$$\sigma = \sigma_0 \left[1 + \frac{H_0^2(t)}{8\pi A} - \frac{H_0^2(x, t)}{8\pi A}\right]^\delta. \tag{2.3}$$

The equation obtained can be simplified by expanding Eq. (2.3) for the electrical conductivity in terms of the small parameter $\varepsilon = (H^2/8\pi A)/(1 + H_0^2 8\pi A)$:

$$\sigma = \sigma_0 (1 + H_0^2/8\pi A)^\delta (1 - \delta\varepsilon). \tag{2.4}$$

In view of the quadratic dependence of $\varepsilon$ on H, for $x > 0$, $\varepsilon$ will decrease rapidly and approach zero at a depth of the order of the skin layer. Therefore, $\varepsilon \ll 1$ over the whole range of integration $0 < x < \infty$, $0 < t < t_0$ except for a small neighborhood near $x = 0$, $t = t_0$, where $\varepsilon$ may be of the order of unity. Consequently, the solution of (2.1), (2.2), and (2.4) for $\varepsilon = 0$ can be used as a first approximation to the solution of the nonlinear equation (2.1)-(2.3). Thus, we obtain

$$\frac{\partial H}{\partial t} = \frac{c^2}{4\pi\sigma_0} (1 + H_0/8\pi A)^{-\delta} \frac{\partial^2 H}{\partial x^2}. \tag{2.5}$$

Introducing the new variable

$$\tau = \int_0^t [1 + H_0^2(\lambda)/8\pi A]^{-\delta} d\lambda, \tag{2.6}$$

we have from (2.5) and (2.2) a linear differential equation whose solution has the form

$$H(x, \tau) = \frac{2}{\sqrt{\pi}} \int_{\xi_1}^{\infty} H_0\left(\tau - \frac{\pi\sigma_0 x^2}{c^2\lambda^2}\right) e^{-\lambda^2} d\lambda, \tag{2.7}$$

where $\xi_1 = (\pi\sigma_0 x^2/c^2\tau)^{1/2}$. A second approximation to the nonlinear equation (2.1)-(2.3) can be constructed by using linear solutions of the type (2.7). Integrating Eq. (2.1) with respect to x from 0 to x:

$$\int_0^x \frac{\partial H}{\partial t} dx = \frac{c^2}{4\pi\sigma} \frac{\partial H}{\partial x} - \frac{c^2}{4\pi\sigma_0} \frac{\partial H}{\partial x}\bigg|_{x=0}.$$

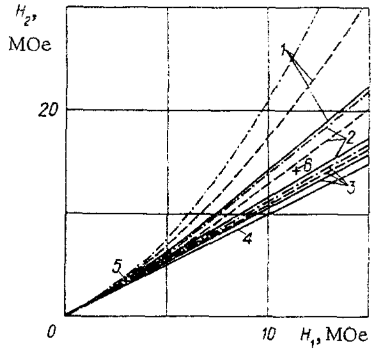


Fig. 3

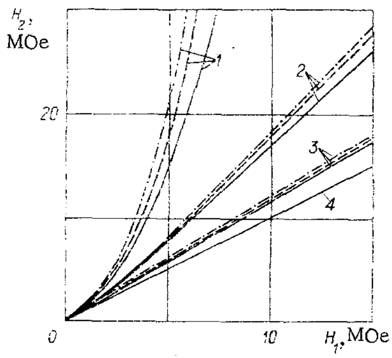


Fig. 4

Taking account of the fact that as $x \to \infty$ $\partial H/\partial x \to 0$, we obtain

$$-\frac{c^2}{4\pi\sigma_0}\frac{\partial H}{\partial x}\bigg|_{x=0} = \int_0^\infty \frac{\partial H}{\partial t}\,dx.$$

Hence for the nonlinear solution (denoted by $H_+$) we have

$$\frac{c^2}{4\pi\sigma}\left|\frac{\partial H_+}{\partial x}\right| = \int_x^\infty \frac{\partial H_+}{\partial t}\,dx, \tag{2.8}$$

where $\sigma$ is given by Eq. (2.3). For the linear solution (denoted by $H_-$) of the diffusion equations (2.1), (2.2) with constant electrical conductivity $\sigma_1$ in the range $\sigma_0 \le \sigma_1 \le \sigma_* = \sigma_0(1 + H_0^2/8\pi A)^\delta$ we obtain the analogous expression

$$\frac{c^2}{4\pi\sigma_1}\left|\frac{\partial H_-}{\partial x}\right| = \int_x^\infty \frac{\partial H_-}{\partial t}\,dx. \tag{2.9}$$

We assume that the linear ($H_-$) and nonlinear ($H_+$) solutions intersect at the point $x_1$. This is shown qualitatively in Fig. 2, where curve 1 is for $H_+$ and curve 2 for $H_-$. Then, subtracting (2.8) from (2.9), we obtain

$$\frac{c^2}{4\pi\sigma_1}\left(\left|\frac{\partial H_-}{\partial x}\right| - \frac{\sigma_1}{\sigma}\left|\frac{\partial H_+}{\partial x}\right|\right)\bigg|_{x=x_1} = \int_{x_1}^\infty \left(\frac{\partial H_-}{\partial t} - \frac{\partial H_+}{\partial t}\right)dx. \tag{2.10}$$

Figure 2 shows that the condition $|\partial H_-/\partial x| < |\partial H_+/\partial x|$ is always satisfied at a given instant at point $x_1$. Taking account of the fact that in the interval $[x_1, \infty]$ $|\partial H_-/\partial t| > |\partial H_+/\partial t|$, we have from (2.10)

$$\left|\frac{\partial H_-}{\partial x}\right| > \frac{\sigma_1}{\sigma}\left|\frac{\partial H_+}{\partial x}\right|.$$

This is possible only when $\sigma > \sigma_1$ at point $x_1$, except for the two points $x_1 = 0$, where $\sigma = \sigma_1 = \sigma_0$, and $x_1 = \infty$, where $\sigma = \sigma_1 = \sigma_*$. This follows from (2.9) by substituting $x_1 = 0$ and $x_1 = \infty$. Thus, taking account of (2.3), we find that on the graph of the field distribution in the medium as a function of depth the nonlinear solution will lie above the transcendental, each point of which is determined by the intersection of the straight line $\eta$ = const ($0 \le \eta \le 1$) with the linear solution (2.7), for which $\tau = \tau_1$, where

$$\tau_1 = \int_0^t \left[1 + \frac{H_0^2(\lambda)}{8\pi A}(1-\eta^2)\right]^{-\delta} d\lambda; \tag{2.11}$$

$$\eta = \frac{2}{\sqrt{\pi}}\frac{1}{H_0}\int_{\xi_2}^\infty H_0\left(\tau_1 - \frac{\pi\sigma_0 x^2}{c^2\lambda^2}\right)e^{-\lambda^2}\,d\lambda, \tag{2.12}$$

where $\xi_2 = (\pi\sigma_0 x^2/c^2\tau_1)^{1/2}$. Since the transcendental solution (2.12) is constructed from linear solutions for which $\sigma \le \sigma_*$, it will lie above the linear solution (2.7) for which $\sigma = \sigma_*$. Thus, we have obtained the second approximation of the nonlinear equation (2.1)-(2.3).

3. For the constant boundary condition $H_0(t) = H_0$ = const, solutions (2.7) and (2.12), respectively, have the form

$$H/H_0 = 1 - \mathrm{erf}\,[\xi(1 + H_0^2/8\pi A)^{\delta/2}]; \quad (3.1)$$

$$\eta = 1 - \mathrm{erf}\left\{\xi\left[1 + \frac{H_0}{8\pi A}(1-\eta^2)\right]^{\delta/2}\right\}, \quad (3.2)$$

where $\xi = (\pi\sigma_0 x^2/c^2 t)^{1/2}$.

The flux skin layer for the linear solution (3.1), the transcendental (3.2), and the numerical solution of (2.1)-(2.3) for $H_0$ = const has the form

$$s_i = \alpha_i^{-1}\sqrt{\frac{c^2 t}{\pi^2\sigma_0}}\left(1 + \frac{H_0^2}{8\pi A}\right)^{-\delta/2} \quad (i = 0, 1, 2),$$

where i = 0 corresponds to the linear solution, i = 1 to the transcendental, and i = 2 to the numerical solution; $\alpha_0 = 1$, $\alpha_1 = s_0/s_1$, $\alpha_2 = s_0/s_2$, where $s_0$, $s_1$, and $s_2$ are determined, respectively, by the linear (3.1), the transcendental (3.2), and the numerical solutions. Thus, for these solutions it can be said that the effect of the magnetic field pressure on the electrical conductivity is reduced to its increase to the value

$$\sigma = \sigma_0\alpha_i^2(1 + H_0^2/8\pi A)^{\delta}.$$

Returning to the original problem, Eq. (1.8) can be written in the form

$$\sigma = \sigma_0(1 + \beta q)^{-1}\alpha_i^2(1 + H_0^2/8\pi A)^{\delta}. \quad (3.3)$$

By introducing the variable $\tau_2 = \tau/\alpha_i^2$, where $\tau$ is given by (2.6), Eqs. (1.3) and (1.4) with the electrical conductivity (3.3) are reduced to a system of equations for the variables x and $\tau_2$ with the electrical conductivity depending only on Joule heat

$$\sigma = \sigma_0(1 + \beta q)^{-1}. \quad (3.4)$$

Thus, while the depth of the skin layer for system (1.3), (1.4), and (3.4) is determined by the function s = s(t), for system (1.3), (1.4), and (3.3) we have $s = s(\tau_2)$. It is known that for a field which is constant on the boundary the skin layer has a depth $s \sim (t/\sigma)^{1/2}$; then the ratio of the corresponding thicknesses of the skin layer will be

$$\frac{s(\tau_2)}{s(t)} = (\tau_2/t)^{1/2} = \alpha_i^{-1}(1 + H_0^2/8\pi A)^{-\delta/2}. \quad (3.5)$$

Substituting Eq. (3.5) into (1.2), we obtain

$$\frac{H_2}{H_1} = \frac{1 + x_*/s(t)}{\alpha_i^{-1}(1 + H_0^2/8\pi A)^{-\delta/2} + x_*/s(t)}. \quad (3.6)$$

Figures 3 and 4 show plots of Eq. (3.6) for copper and lead shells, respectively, for the following cases: 1) $x_* = 0$; 2) $x_* = s(t)$; 3) $x_* = 5s(t)$; 4) $x_* \gg s(t)$; 5) data from [2]; 6) data from [3]; the dash–dot curves correspond to the linear solutions, the dashed curves to the transcendental, and the solid curves to the numerical solutions. The graphs show that for a field H = 15 MOe the maximum differences between the linear and transcendental solutions and the numerical solution occur at $x_* = 0$, and are 80 and 40%, respectively, for a copper shell, and 133 and 65% for a lead shell. These differences decrease with increasing $x_*$, and for $x_* \approx s(t)$ are, respectively, 25 and 13% for copper, and 9 and 6% for lead. Thus, Eq. (3.6) for the linear and transcendental solutions can be used to estimate the effect of pressure on the magnitude of the magnetic field in the liner.

Figures 3 and 4 show that for $x_* \gg s(t)$ $H_1 = H_2$, and in calculating the maximum attainable magnetic field the effect of pressure on the electrical conductivity of the medium can be neglected, independently of the magnitude of the field intensity. For distances to the reversal point which are smaller than or comparable with the depth of the thermal skin layer [$x_* \lesssim s(t)$], the pressure dependence of the electrical conductivity of the shell significantly affects the magnitude of the magnetic field for a copper liner in the range H > 3-5 MOe, and for a lead liner in the range H > 1-2 MOe. Therefore, calculations for shells of these metals in the indicated ranges of field intensities which take account of the dependence of the electrical conductivity only on Joule heat will underestimate the magnitude of the maximum attainable field, and the more so the shorter the distance to the reversal point in comparison with the depth of the thermal skin layer. Points 5 and 6 of Fig. 3 show that the relations obtained are in good agreement with experiment. The experimental values of the field are plotted along the axis of ordinates, and the corresponding calculated values of $H_1$ along the axis of abscissas.

## LITERATURE CITED

1. Yu. D. Bakulin, V. F. Kuropatenko, and A. V. Luchinskii, "MHD calculations for exploding wires," Zh. Tekh. Fiz., 46, No. 9 (1976).

2. N. N. Kalitkin and L. S. Tsareva, "Approximate theory for amplification of magnetic fields," Zh. Tekh. Fiz., 39, No. 8 (1969).
3. R. E. Kidder, "Compression of magnetic field inside a hollow explosive-driven cylindrical conductor," in: Proc. of Conf. on Megagauss Magnetic Fields, Frascati, Italy (1965).
4. H. Knoepfel, Pulsed High Magnetic Fields, American Elsevier, New York (1970).
5. N. F. Mott and H. Jones, The Theory of the Properties of Metals and Alloys, Dover, New York (1958).
6. I. K. Kikoin (ed.), Tables of Physical Quantities. Handbook [in Russian], Nauka, Moscow (1976).
7. K. P. Stanyukovich (ed.), Physics of an Explosion [in Russian], Nauka, Moscow (1975).

# PHENOMENOLOGICAL MODEL OF PUNCH-THROUGH

S. T. Mileiko and O. A. Sarkisyan

UDC 539.3.534.1

At this time the most widespread method for analyzing the punch-through process, based on a numerical procedure for solving the problem posed under any assumptions about the constitutive equations of the medium and, as a rule, without considering fracture processes (e.g., [1]), yields results that are not always convenient from the practical viewpoint. In particular, these results are difficult to compare with experimental data in which the critical punch-through velocities are most often recorded (e.g., [2, 3]). In the probable future, when crack generation and development criteria and the constitutive equations of a medium become sufficiently reliable under high-speed loading conditions, numerical methods will permit the efficient solution of practical problems. However, at this time there is a need for simple models to describe the punch-through process. The model proposed is an example of this kind of phenomenology.

It is shown in [2] that in those cases when punch-through is accompanied by the recovery of a "plug," the main contribution to the resistance against inserting the impactor is from plastic deformation or brittle fragmentation of a comparatively thin cylindrical layer. In this case, at least if plastic deformation occurs, it is evident that the resistance to the impactor motion should depend on the velocity v of the impactor.

Let us assume that the quantity F for a given impactor-obstacle pair depends only on v. (It is clear that this assumption is invalid when the impactor is near the obstacle surface, hence, we consider only obstacles of sufficiently great thickness.) Let us approximate this dependence by a power-law function

$$F(v) = -Kv^n,$$

where K and n are constants.

If $v = v_0$ for $x = 0$ (x is the coordinate in the direction of impactor motion and the origin is on the frontal surface of the obstacle), then

$$v_0^{2-n} - v^{2-n} = k(2-n)x,$$

where $k = K/m$ and m is the impactor mass.

For an obstacle thickness h we have the critical velocity $v_0 = v_*$ of the impactor so that $v = 0$ for $x = h$:

$$v_* = k(2-n)h.$$

For $v_0 > v_*$ we have the velocity $v_1$ of impactor taking off from the obstacle so that

$$\tilde{v}_0^{2-n} - \tilde{v}_1^{2-n} = 1,$$

where $\tilde{v} = v/v_*$.

The result obtained (which corresponds, for n = 0, to the condition of constancy of the energy absorbed by the obstacle, and is sometimes [4] taken as being sufficiently evident) turns out to be wonderfully simple: the curve $\tilde{v}_1(\tilde{v}_0)$ is independent of the obstacle thickness. This simplicity requires convincing experimental confirmation.

Chernogolovka. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 140-142, September-October, 1981. Original article submitted July 24, 1980.